M3 - January 2006

1- a) $\rightarrow F = T\sin 60 = \frac{\sqrt{3}}{2}T$

$\uparrow T\cos 60 = 0.8g$

$T = 1.6g$

$F = \frac{1.6g\sqrt{3}}{2} = 13.6N\ (3sf)$

b) $T = \frac{\lambda x}{a}$

$1.6g = \frac{24x}{1.2}$

$x = \frac{1.2 \times 1.6g}{24}$

$= 0.784m\ (3sf)$

c) $E = \frac{\lambda x^2}{2a}$

$= \frac{24 \times 0.784^2}{2 \times 1.2}$

$= 6.15J\ (3sf)$

2- a) $a = 2\sin\frac{t}{2}$

b) $\frac{dx}{dt} = 8 - 4\cos\frac{t}{2}$

$\int_{x_1}^{x_2} = \int_0^{\pi/2}$

$\frac{dv}{dt} = 2\sin\frac{t}{2}$

$\int_4^v dv = 2\int_0^t \sin\frac{t}{2}\,dt$

$v - 4 = 4\left[-\cos\frac{t}{2}\right]_0^t$

$v = 4 - 4\cos\frac{t}{2} + 4$

$= 8 - 4\cos\frac{t}{2}$

$\int_{x_1}^{x_2} dx = \int_0^{\pi} \left(8 - 4\cos\frac{t}{2}\right) dt$

$x_2 - x_1 = 4\left[2t - 2\sin\frac{t}{2}\right]_0^{\pi}$

$= 8\left(\frac{\pi}{2} - \frac{\sqrt{2}}{2}\right)$

$= 4(\pi - \sqrt{2})\,m$

3- $\left[F = ma\right]$

$-\frac{cm}{x^2} = m\frac{v\,dv}{dx}$

$c\int_R^x -x^{-2}\,dx = \int_u^v v\,dv$

$c\left[+x^{-1}\right]_R^x = \frac{1}{2}\left[v^2\right]_u^v$

$c\left(\frac{1}{x} - \frac{1}{R}\right) = \frac{1}{2}v^2 - \frac{1}{2}u^2$

$v^2 = u^2 + 2c\left(\frac{1}{x} - \frac{1}{R}\right)$

b) $\frac{1}{2}\cdot m\cdot\left(u^2 + \frac{2c}{2R} - \frac{2c}{R}\right) = \frac{1}{2}\cdot\frac{1}{2}\cdot m\cdot\left(u^2 + \frac{2c}{R} - \frac{2}{R}\right)$

$2u^2 - \frac{2c}{R} = u^2$

$u^2 R = 2c$

$c = \frac{u^2 R}{2}$

4- a) $\frac{h}{2} \cdot 3m + \left(h + \frac{3r}{8}\right) \cdot 2m = 0\bar{G} \cdot 5m$

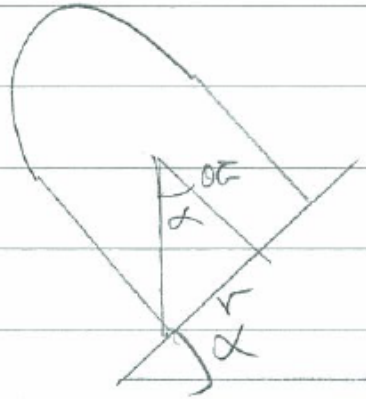
$$\frac{3h}{2} + 2h + \frac{3r}{4} = 5\,0\bar{G}$$

$$\frac{7h}{2} + \frac{3r}{4} = 0\bar{G} \times 5$$

$$\frac{7h}{10} + \frac{3r}{20} = 0\bar{G}$$

$$0\bar{G} = \frac{14h + 3r}{20}$$

b)



$$\tan x = \frac{4}{3} = \frac{r}{0\bar{G}}$$

$$\frac{4}{3} = \frac{20r}{14h + 3r}$$

$$60r = 56h + 12r$$

$$48r = 56h$$

$h = \frac{48r}{56} = \frac{6r}{7}$

5. a) $\uparrow mg = T$

$T = \frac{\lambda x}{a}$

$mg = \frac{\lambda \cdot l}{4 \cdot l} = \frac{\lambda}{4}$

$\lambda = 4mg$

b) $T = \frac{\lambda x}{a} = \frac{\lambda\left(\frac{l}{4} + x\right)}{l}$

$= \frac{mgl + 4mgx}{l}$

$= mg + \frac{4mgx}{l}$

$\downarrow [F = ma]$

$mg - T = m\frac{d^2x}{dt^2}$

$mg - mg - \frac{4mgx}{l} = m\frac{d^2x}{dt^2}$

$\frac{d^2x}{dt^2} = -\frac{4gx}{l}$

c) $a = \frac{l}{2}$

$v^2 = w^2(a^2 - x^2)$

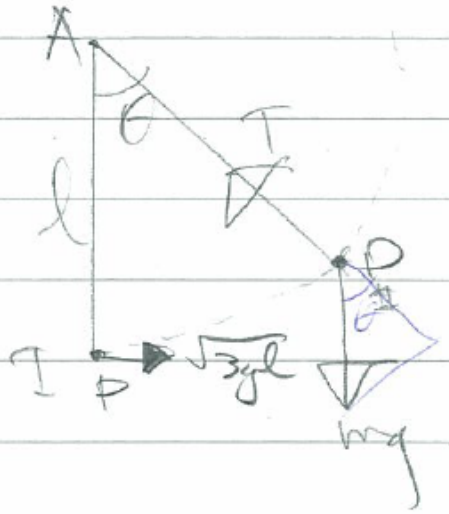
$= \frac{4g}{l}\left(\frac{l^2}{4} - \frac{l^2}{16}\right)$

$= \frac{12gl}{16} = \frac{3gl}{4}$

$v = \frac{1}{2}\sqrt{3gl}$

d) moves freely under gravity then SHM between B...

6-



a) $ME_I = ME_{II}$

$$\frac{1}{2}\cdot m\cdot 3gl = \frac{1}{2}\cdot m\cdot v^2 + mg(l - l\cos\theta)$$

$$3gl = v^2 + 2gl - 2gl\cos\theta$$

$$v^2 = gl + 2gl\cos\theta$$

$[F = ma]\uparrow$

$$T - mg\cos\theta = \frac{m}{l}(gl + 2gl\cos\theta)$$

$$T - mg\cos\theta = mg + 2mg\cos\theta$$

$$T = mg + 3mg\cos\theta = mg(1 + 3\cos\theta)$$

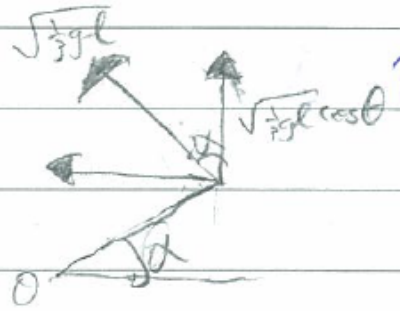
b) $0 = mg(1 + 3\cos\theta)$

$$-\frac{1}{3} = \cos\theta$$

$$v^2 = gl + 2gl\cdot -\frac{1}{3}$$

$$= gl - \frac{2}{3}gl = \frac{gl}{3}$$

$$v = \sqrt{\frac{1}{3}gl}$$

c) 

$\uparrow [v^2 = u^2 + 2as]$

$0 = \frac{1}{3}gl \cdot \frac{8}{9} - 19.6s$

$s = \frac{8gl}{27 \cdot 2g} = \frac{4l}{27}$

$H = l + \frac{4l}{27} + l\sin\alpha = \frac{31l}{27} + \frac{l}{3} = \frac{40l}{27}$

7- a) $\leftarrow [F = ma]$

$T\cos 30 = m \cdot \frac{kg}{3\alpha} \cdot 2\alpha\cos 30$

$T = \frac{2kmg}{3}$

b) $\uparrow T\sin 30 + N = mg$

$\frac{kmg}{3} + N = mg$

$N = mg\left(1 - \frac{k}{3}\right)$

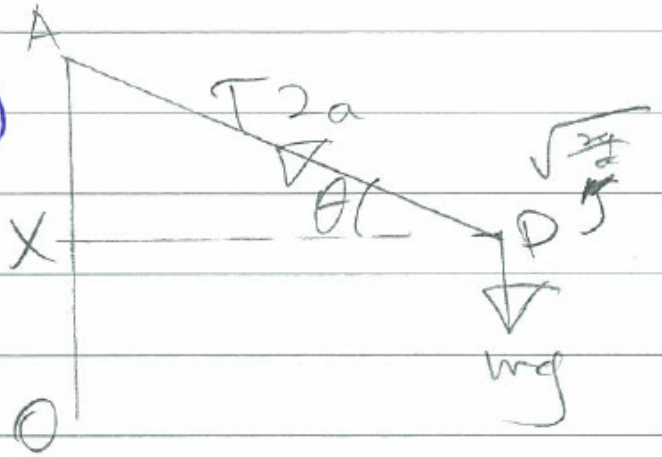
c) $N \geq 0$

$mg\left(1 - \frac{k}{3}\right) \geq 0$

$1 \geq \frac{k}{3}$

$k \leq 3$

d)



$\leftarrow [F = ma]$

$T\cos\theta = m\frac{2g}{a}\,2a\cos\theta$

$T = 4mg$

$\uparrow T\sin\theta = mg$

$4mg\sin\theta = mg$

$\sin\theta = \frac{1}{4}$

$OA = 2a\sin 30 = a$

$OX = 2a\sin\theta = \frac{2a}{4} = \frac{a}{2}$

$\overrightarrow{OX} = \frac{2\underline{a}}{2} - \underline{a} = \overrightarrow{OA}$

$\therefore$ X is the mid-point of OA